REVISION ONE (Solutions)

Year 11 Examination

Question/Answer Booklet

MATHEMATICS METHODS UNITS 1 AND 2

Section One: Calculator-free

Time allowed for this section

Reading time before commencing work: five minutes
Working time for this section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Section One: Calculator-free **(51 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

Calculate the value of

(a) $16^{-0.5}$. (2 marks)

$$16^{-0.5} = \frac{1}{\sqrt{16}} = \frac{1}{4}$$

(b) $(a \div b)^2$ when $a = 4 \times 10^2$ and $b = 8 \times 10^3$, leaving your answer in scientific notation. (3 marks)

$$\left(\frac{4 \times 10^2}{8 \times 10^3}\right)^2 = \left(0.5 \times 10^{-1}\right)^2$$

$$= 0.25 \times 10^{-2}$$

$$= 2.5 \times 10^{-3}$$

Question 2 **(9 marks)**

(a) Determine $\frac{dy}{dx}$ for

(i) $y = \frac{4x^4}{3}$. (1 mark)

$$\boxed{\frac{dy}{dx} = \frac{16x^3}{3}}$$

(ii) $y = \frac{12}{\sqrt{x}}$. (2 marks)

$$y = 12x^{-0.5}$$

$$\frac{dy}{dx} = -6x^{-1.5}$$

(b) Determine $f'(2)$ if $f(x) = \frac{x^2}{4} - \frac{4}{x}$. (3 marks)

$$f'(x) = \frac{x}{2} + \frac{4}{x^2}$$

$$\begin{aligned} f'(2) &= 1 + 1 \\ &= 2 \end{aligned}$$

(c) Determine $g(x)$ if $g(1) = -1$ and $g'(x) = 2x^2 + \frac{2x}{3} + 5$. (3 marks)

$$g(x) = \frac{2x^3}{3} + \frac{x^2}{3} + 5x + c$$

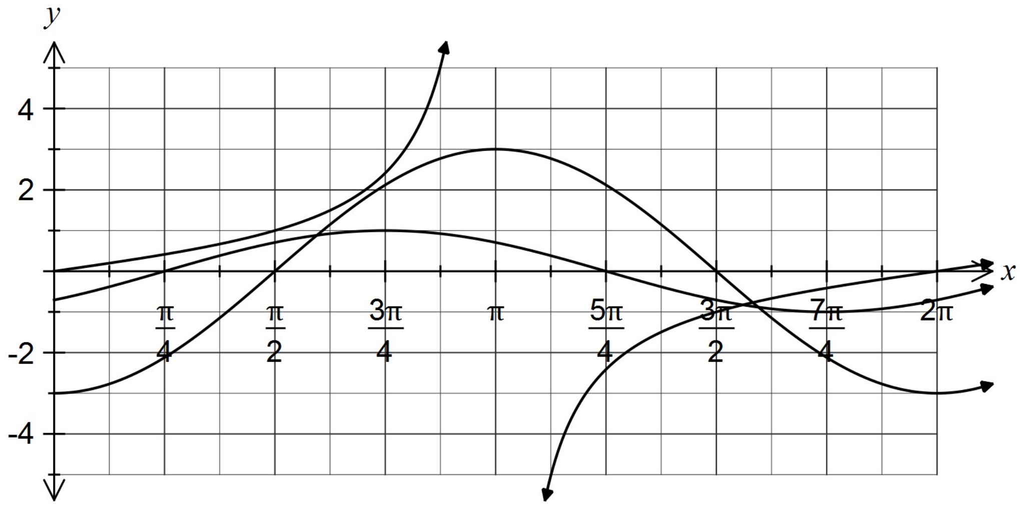
$$-1 = \frac{2}{3} + \frac{1}{3} + 5 + c$$

$$c = -7$$

$$g(x) = \frac{2x^3}{3} + \frac{x^2}{3} + 5x - 7$$

Question 3 **(6 marks)**

(a) The graphs of $y = \tan(ax)$, $y = b\cos(x)$ and $y = \sin(x + c)$ are shown below.



Determine the values of the constants a, b and c. (3 marks)

$$a = \frac{1}{2} \qquad b = -3 \qquad c = -\frac{\pi}{4}$$

(b) Solve the equation $\sqrt{3}\cos\left(x - \frac{\pi}{2}\right) = \cos(x)$ for $0 \le x \le 2\pi$. (3 marks)

$$\sqrt{3}\cos\left(x - \frac{\pi}{2}\right) = \cos(x)$$

$$\sqrt{3}\sin(x) = \cos(x)$$

$$\frac{\sin(x)}{\cos(x)} = \tan(x) = \frac{1}{\sqrt{3}}$$

$$x = \frac{\pi}{6}, \frac{7\pi}{6}$$

Question 4 **(7 marks)**

(a) Evaluate $x^{2a} \cdot x^b$ when $x = 64$, $a = 2$ and $b = -4.5$. (3 marks)

Solution
$x^{2a+b} = 64^{-\frac{1}{2}}$ $= \frac{1}{\sqrt{64}}$ $= \frac{1}{8}$
Specific behaviours
✓ eliminates negative indices   ✓ eliminates fractional indices   ✓ states value

(b) The first two terms of a geometric sequence are 3×10^{-4} and 6×10^{-6}. Calculate the fifth term of the sequence, giving your answer in scientific notation. (4 marks)

Solution
$r = \frac{6 \times 10^{-6}}{3 \times 10^{-4}} = 2 \times 10^{-2}$ $T_4 = (3 \times 10^{-4})(2 \times 10^{-2})^4$ $= 3 \times 10^{-4} \times 2^4 \times 10^{-8}$ $= 48 \times 10^{-12}$ $= 4.8 \times 10^{-11}$
Specific behaviours
✓ evaluates ratio   ✓ indicates expression for 4th term   ✓ simplifies   ✓ expresses term in scientific notation

Question 5 **(9 marks)**

Solve the following equations for x:

(a) $(x-11)^2-49=0$. (2 marks)

Solution
$x-11=\pm 7$ $x=4, 18$
Specific behaviours
✓ adjusts equation and takes square root ✓ states both solutions

(b) $27^{x+1}=9^{1-x}$. (3 marks)

Solution
$3^{3(x+1)}=3^{2(1-x)}$ $3x+3=2-2x$ $x=-\dfrac{1}{5}$
Specific behaviours
✓ writes both sides as powers of 3 ✓ equates indices ✓ solves

(c) $\sin^2 x-\cos^2 x=\frac{1}{2}, 0\le x\le 360°$. (4 marks)

Solution
$\sin^2 x-(1-\sin^2 x)=\dfrac{1}{2}$ $\sin^2 x=\dfrac{3}{4}$ $\sin x=\dfrac{\sqrt{3}}{2}\Rightarrow x=60, 120$ $\sin x=-\dfrac{\sqrt{3}}{2}\Rightarrow x=240, 300$ $x=60°, 120°, 240°, 300°$
Specific behaviours
✓ uses Pythagorean identity and simplifies ✓ shows two possible values for $\sin x$ ✓ determines first two solutions ✓ determines another two solutions

Question 6 **(5 marks)**

(a) Determine $f'(x)$ if

(i) $f(x) = 5x^4 + x$. (1 mark)

Solution
$f'(x) = 20x^3 + 1$
Specific behaviours
✓ differentiates

(ii) $f(x) = (2x + 3)^2$. (2 marks)

Solution
$f(x) = 4x^2 + 12x + 9$ $f'(x) = 8x + 12$
Specific behaviours
✓ expands ✓ differentiates

(b) The area of an oil slick, at time t hours, is given by $A(t) = 0.5t^3 - 2t^2 + 7$ square meters. Determine the instantaneous rate of change of the area of the slick when $t = 10$ hours.

(2 marks)

Solution
$A'(t) = 1.5t^2 - 4t$ $A'(10) = 150 - 40 = 110 \text{ m}^2/\text{h}$
Specific behaviours
✓ differentiates correctly ✓ substitutes and simplifies

Question 7 **(10 marks)**

(a) Expand $(x-2)^4$. (3 marks)

$$(x-2)^4 = x^4 + 4x^3(-2) + 6\times x^2(-2)^2 + 4\times x(-2)^3 + (-2)^4$$

$$= x^4 - 8x^3 + 24x^2 - 32x + 16$$

(b) Solve the following for x:

(i) $4^{2x-1} = \frac{1}{8}$. (3 marks)

$$LHS = 2^{2(2x-1)}$$

$$RHS = 2^{-3}$$

$$4x - 2 = -3$$

$$x = -\frac{1}{4}$$

(ii) $x^3 - x^2 - 17x - 15 = 0$. (4 marks)

$$x = -1, \quad -1 - 1 + 17 - 15 = 0$$

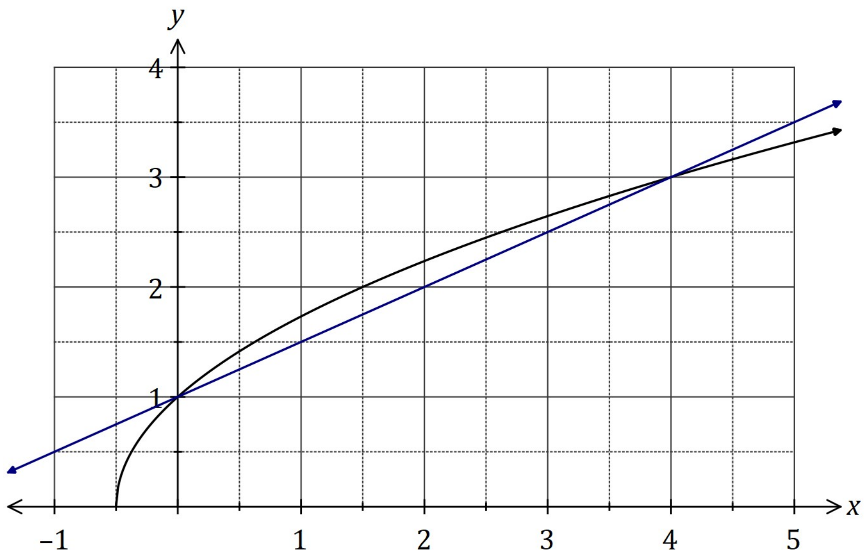
$$x^3 - x^2 - 17x - 15 = (x+1)(x^2 - 2x - 15)$$

$$= (x+1)(x+3)(x-5)$$

$$x = -1, \quad -3, \ 5$$

Question 8 **(5 marks)**

The graph of $y = f(x)$ is shown below, where $f(x) = \sqrt{2x+1}$.



The difference quotient is shown here:

$$\frac{f(x+h) - f(x)}{h}$$

(a) Add to the graph a secant whose slope represents the difference quotient when $x = 0$ and $h = 4$, and state the value of this sl marks)

Solution
See graph - slope is $\frac{1}{2}$
Specific behaviours
✓ secant through (0, 1) and (4, 3)
✓ correct slope

(b) Evaluate the difference quotient as $h \to 0$ to determine the slope of $f(x)$ when $x = 0$. (3 marks)

Solution

$$\begin{aligned}
\frac{f(x+h) - f(x)}{h} &= \frac{\sqrt{0+2h+1} - \sqrt{0+1}}{h} \\
&= \frac{\sqrt{2h+1} - 1}{h} \times \frac{\sqrt{2h+1}+1}{\sqrt{2h+1}+1} \\
&= \frac{2h}{h\left(\sqrt{2h+1}+1\right)} \\
&= \left.\frac{2}{\sqrt{2h+1}+1}\right|_{h=0} \\
&= 1
\end{aligned}$$

Specific behaviours

✓ substitutes $f(x)$, $x = 0$ and simplifies
✓ removes surd from numerator
✓ substitutes $h = 0$ and simplifies